**Home Assignment**

**Numerical differentiation**

1. Estimate the first derivative of function using central differences:

at *x* = 0.5 employing step sizes of *h*1 = 0.5 and *h*2 = 0.25. Then compute an improved estimate with Richardson extrapolation. Recall that the true value is −0.9125. Estimate the true percent relative error **εt** for each approximation.

1. Compute forward and backward difference approximations and central difference approximations for the first derivative of ***y=sin(x)*** at ***x=π/4*** using a value of ***h = π/12***. Estimate the true percent relative error **εt** for each approximation.
2. Compute forward and backward difference approximations and central difference approximations for the first derivative of ***y=log(x)*** at ***x=25*** using a value of ***h = 2***. Estimate the true percent relative error **εt** for each approximation.
3. The following data was collected for the distance traveled versus time for a rocket:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| t, s | 0 | 25 | 50 | 75 | 100 | 125 |
| y, km | 0 | 32 | 58 | 78 | 92 | 100 |

Use numerical differentiation to estimate the rocket’s velocity and acceleration at each time.

1. Develop a user-friendly program to obtain first-derivative estimates for unequally spaced data. Test it with the following data:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| x | 1 | 1.5 | 1.6 | 2.5 | 3.5 |
| f(x) | 0.6767 | 0.3734 | 0.3261 | 0.08422 | 0.01596 |

where . Compare your results with the true derivatives.

1. The following data is provided for the velocity of an object as a function of time,

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| t, s | 0 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 |
| v, m/s | 0 | 34.7 | 61.8 | 82.8 | 99.2 | 112.0 | 121.9 | 129.7 | 135.7 | 140.4 |

(a) Using the best numerical method available, how far does the object travel from ***t = 0*** to ***28 s***?

(b) Using the best numerical method available, what is the object’s acceleration at ***t = 28 s***.

(c) Using the best numerical method available, what is the object’s acceleration at t = 0 s.